

Nucleon Structure with clover-Wilson Fermions

LHP & NME proposal

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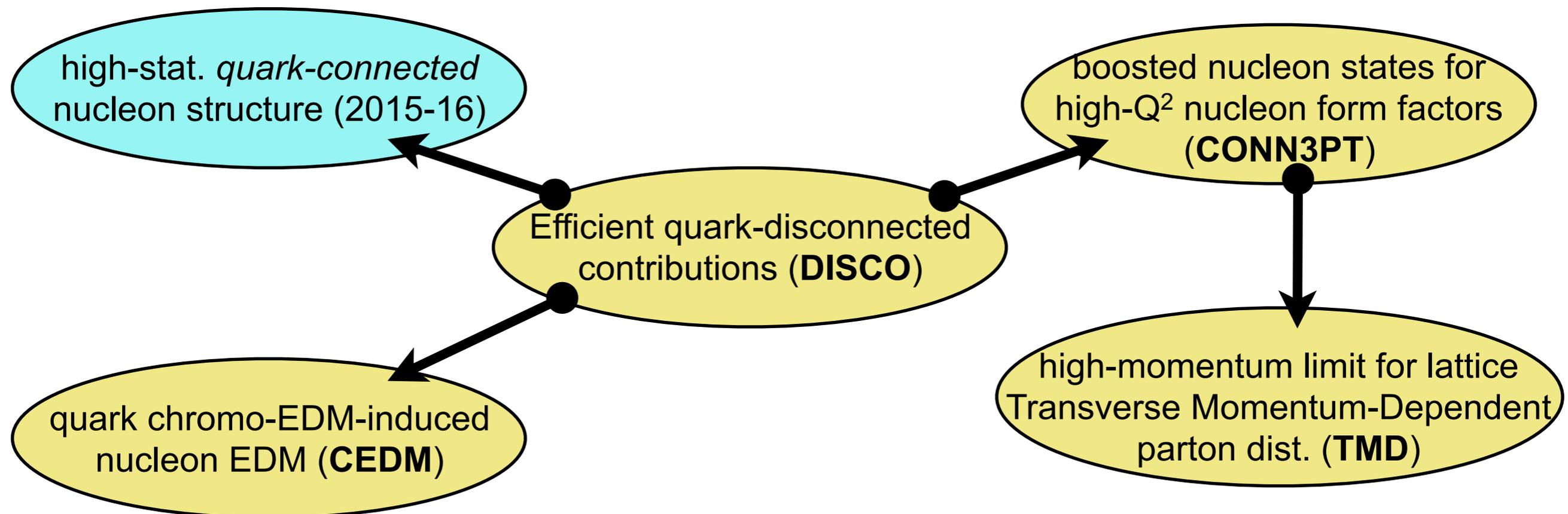
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Nucleon Structure with Isotropic Wilson Lattices

Goal : Compute Nucleon Structure and Quark Matrix Elements with high statistical precision and robust control of systematic errors

Four interconnected components :

- **DISCO**: disconnected diagrams with Hierarchical Probing and Deflation [A.Gambhir, K. Orginos]
- **CONN3PT** : Nucleon form factors with high momentum transfer using boosted nucleon operators [orig. proposed by B.Musch]
- **CEDM** : Nucleon electric dipole moment induced by quark chromo-EDM CP-violating operator
- **TMD** : Transverse-momentum dependent parton distributions with high-momentum limit



Isotropic clover-Wilson Lattices, Present&Outlook

(ensembles selected for analysis)

ID	a[fm]	Volume	m_π	$m_\pi L$	Traj. available	
C13	0.114	32 ³ x96	300	5.6	10,000	DISCO,C3PT, CEDM
D5	0.081	32 ³ x64	312	4.0	5,000	DISCO,C3PT, CEDM, TMD
D6	0.080	48 ³ x96	192	3.7	2,500	DISCO,C3PT, CEDM
D7	0.080	64 ³ x128	192	4.9	2,000	generation continuing
D8	0.080	72 ³ x196	400 → 140	4.1	thermalizing	Bluewaters' 1st Y
D9	0.080	96 ³ x256	140	5.4	2k planned	Bluewaters' 2nd Y

Efficient Calculation of Disconnected Diagrams

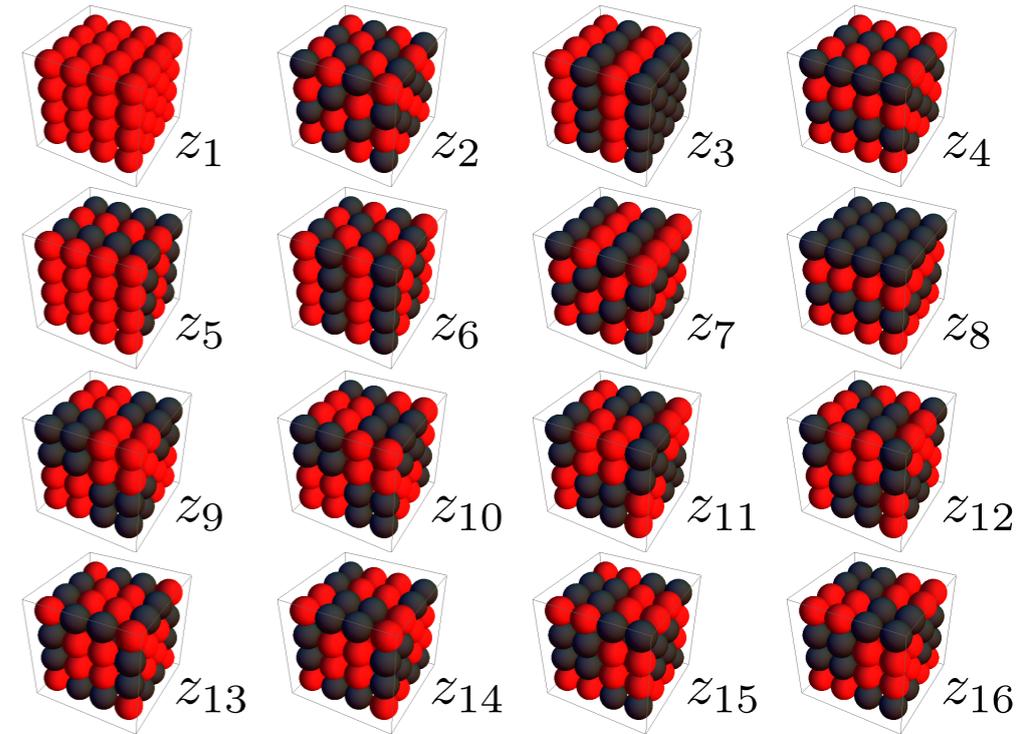
Hierarchical probing [K.Orginos, A.Stathopoulos, '13] :

In sum over 2^{dk+1} vectors (d=3),
 $\text{dist}(x,y) \leq 2^k$ terms cancel exactly:

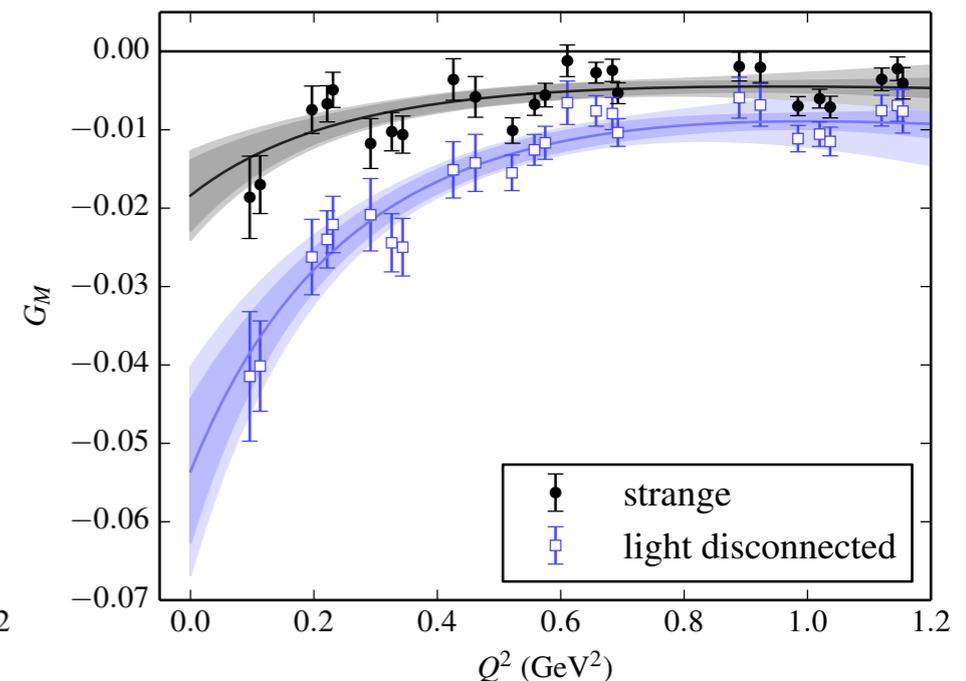
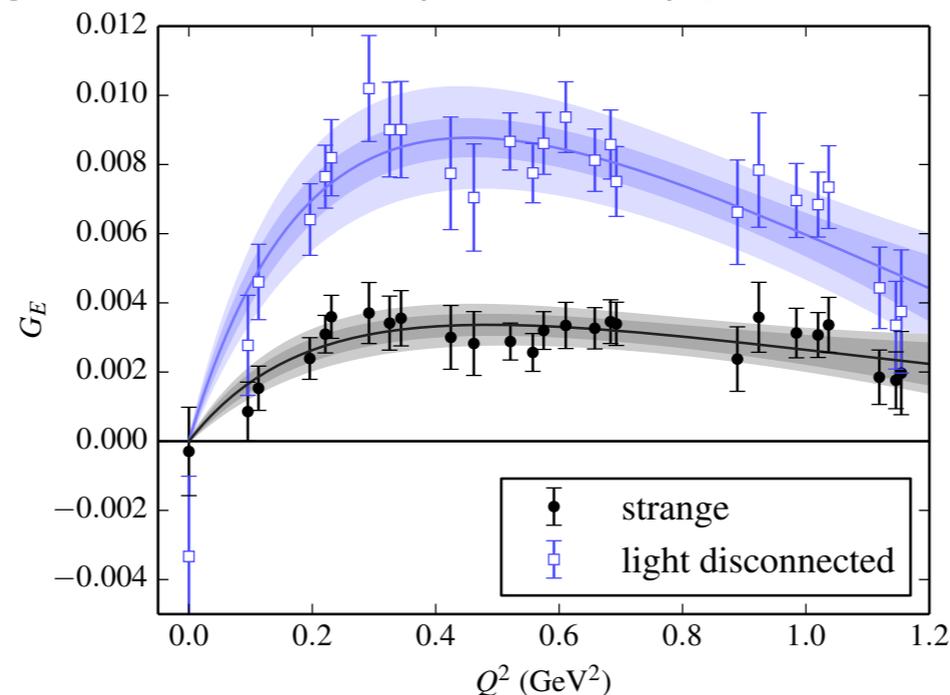
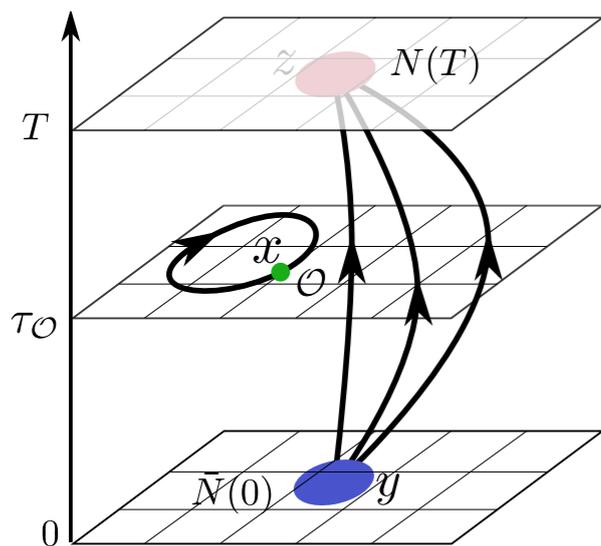
$$1 \leq \sum_a |x_a - y_a| \leq 2^k : \quad \frac{1}{N} \sum_i z_i(x) z_i(y)^\dagger \equiv 0$$

$$z_i \xrightarrow{a} z_i \odot \xi, \quad \xi(x) = \text{random } Z_2\text{-vector}$$

- NEW: reduce variance by treating low modes of $(\not{D}^\dagger \not{D})$ exactly [K.Orginos, A.Gambhir]

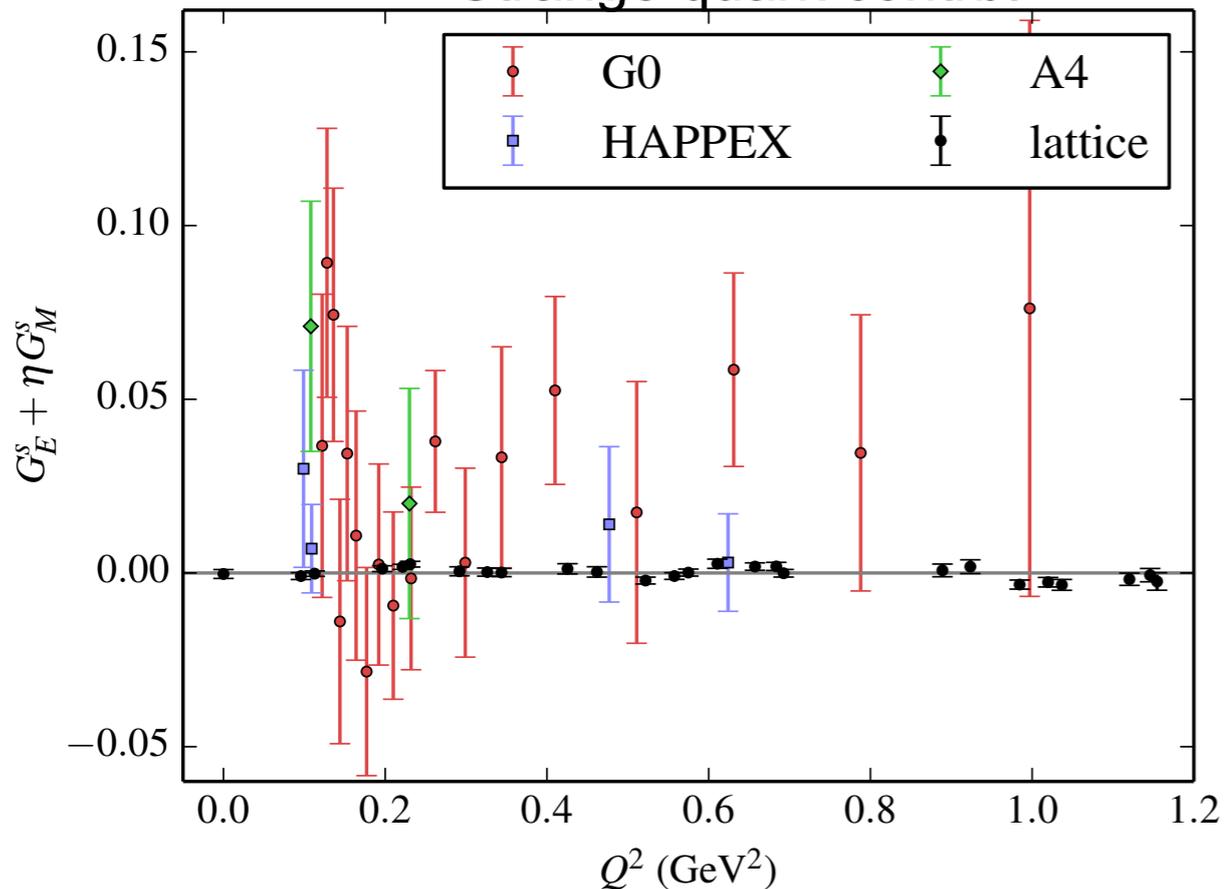


Disconnected light & strange form factors (300 MeV) [J.Green, S.Meinel's, et al, PRD92:031501]



Strange Quark Contributions, Lattice vs Exp.

elastic $e-p$ scattering asymmetry
 \sim Strange quark contrb.



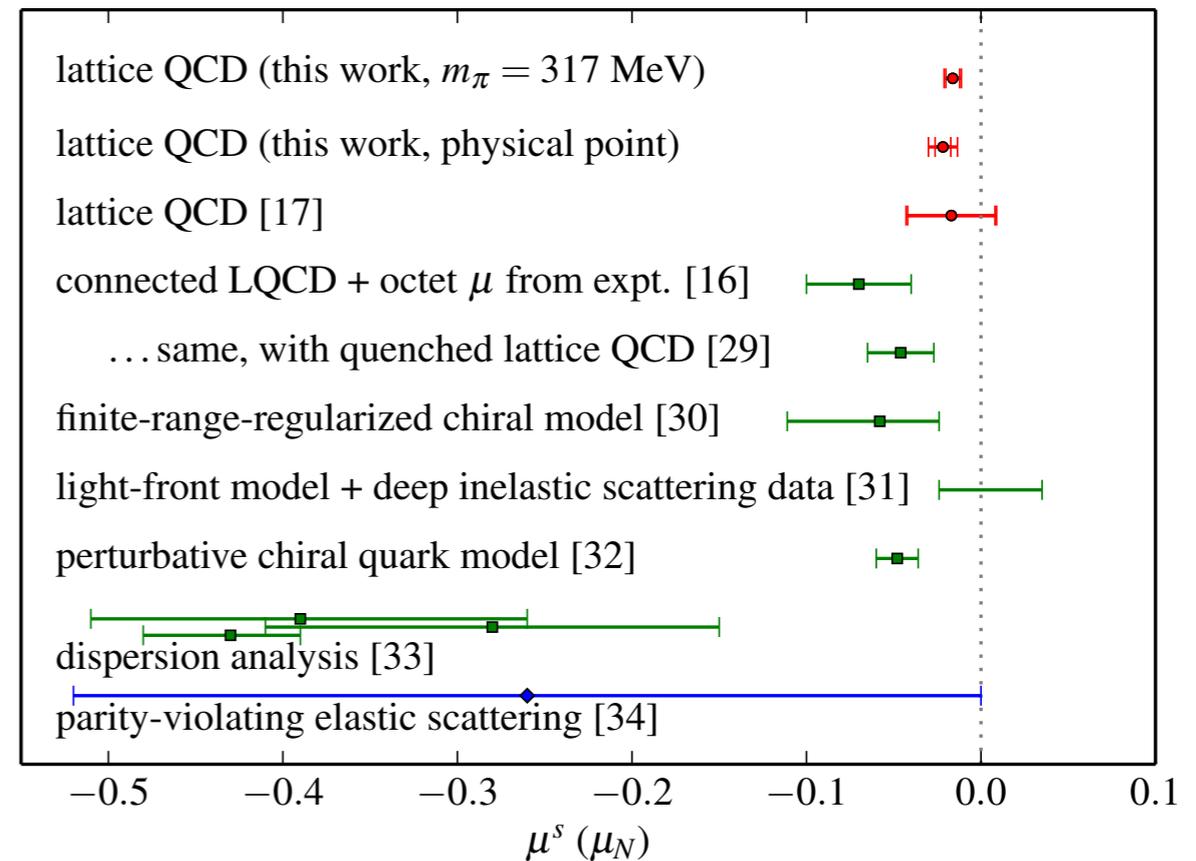
HAPPEX, G0, A4 data
[\[PRL108:102001\(2012\)\]](#)

vs.

Lattice QCD ($m_\pi = 317$ MeV)
[\[J. Green, S. Meinel; PRD92:031501\]](#)

(Lattice "kinematic factor" $\eta = \frac{Q^2}{0.94 \text{ GeV}^2}$)

strange quark magnetic moment



PQChPT-inspired linear extrapolation in
 $(m_{\text{loop}})^2 \sim (m_{\text{light}} + m_{\text{disc}})$ to phys.point

$$(r_E^2)^s = -0.00535(89)(56)(113)(20) \text{ fm}^2$$

$$(r_M^2)^s = -0.0147(61)(28)(34)(5) \text{ fm}^2$$

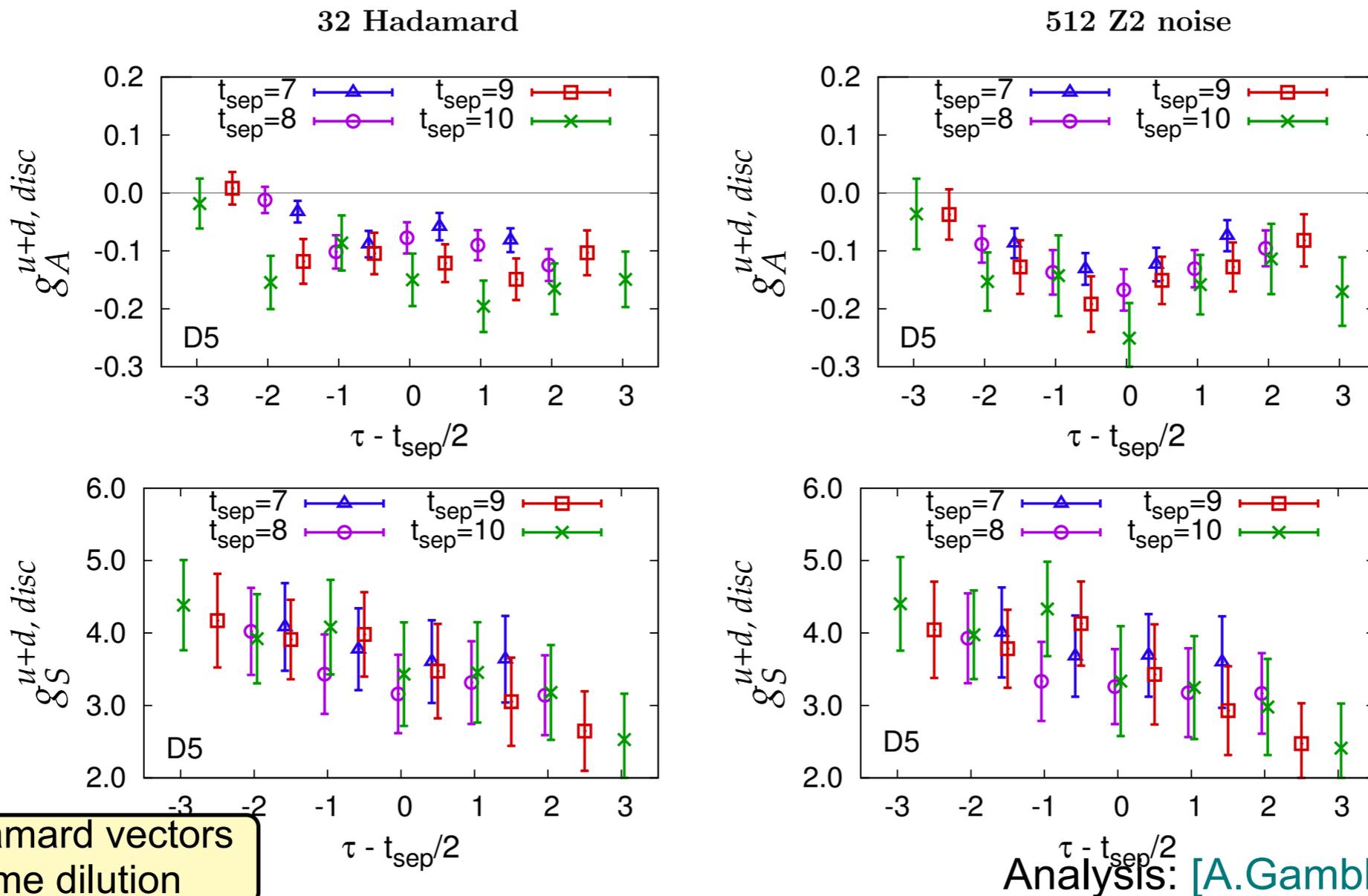
$$\mu^s = -0.0184(45)(12)(32)(1) \mu_N^{\text{lat}}$$

Errors = (statistic)(fit)(exc.state)(discr.)

Deflated Hierarchical Probing

- Variance of HP estimator $\sum_{x \neq y} |\mathbb{D}^{-1}(x, y)|^2$ comes long-distance low modes $|\mathbb{D}^{-1}(x, y)|^2 \sim e^{-m_\pi|x-y|}$
- [A.Gambhir, K.Orginos, A.Stathopoulos] : augmenting the HP estimator by treating the low modes of $(\mathbb{D}^\dagger \mathbb{D})$ exactly

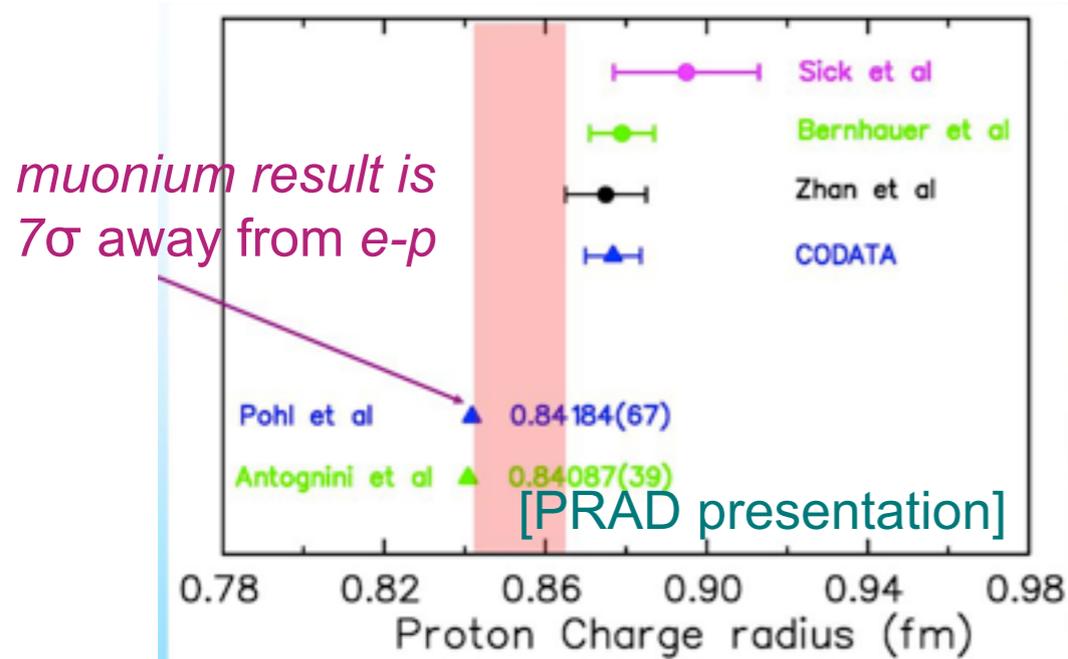
GPU-ready



4D Hadamard vectors
NO time dilution

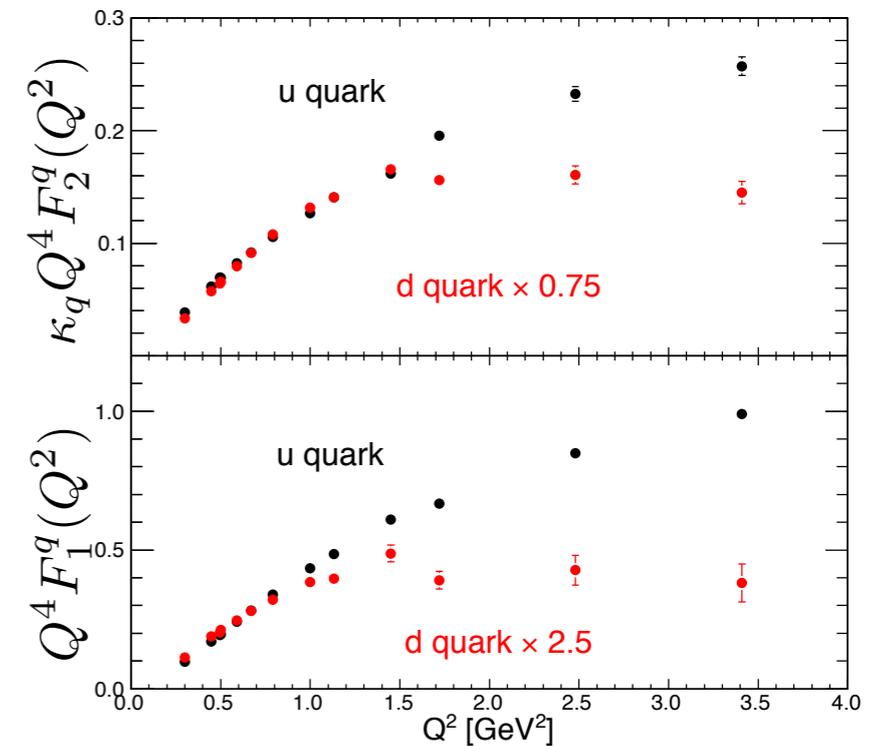
Nucleon Form Factors in Experiments

- Proton radius: 7σ difference; JLab pRAD, MUSE ($e^\pm, \mu^\pm - p$)

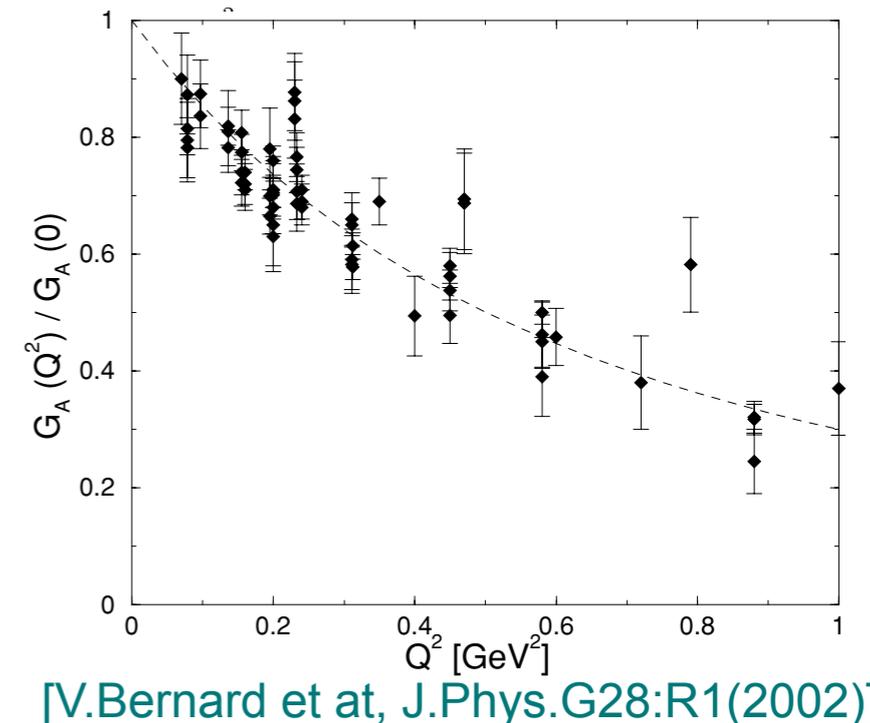


- Form Factors at high momentum: JLab@12GeV : $Q^2 \rightarrow \infty$ scaling; flavor separation

u & d contributions to $F_{1,2}$ form factors [G.D.Cates et al., PRL 106:252003]

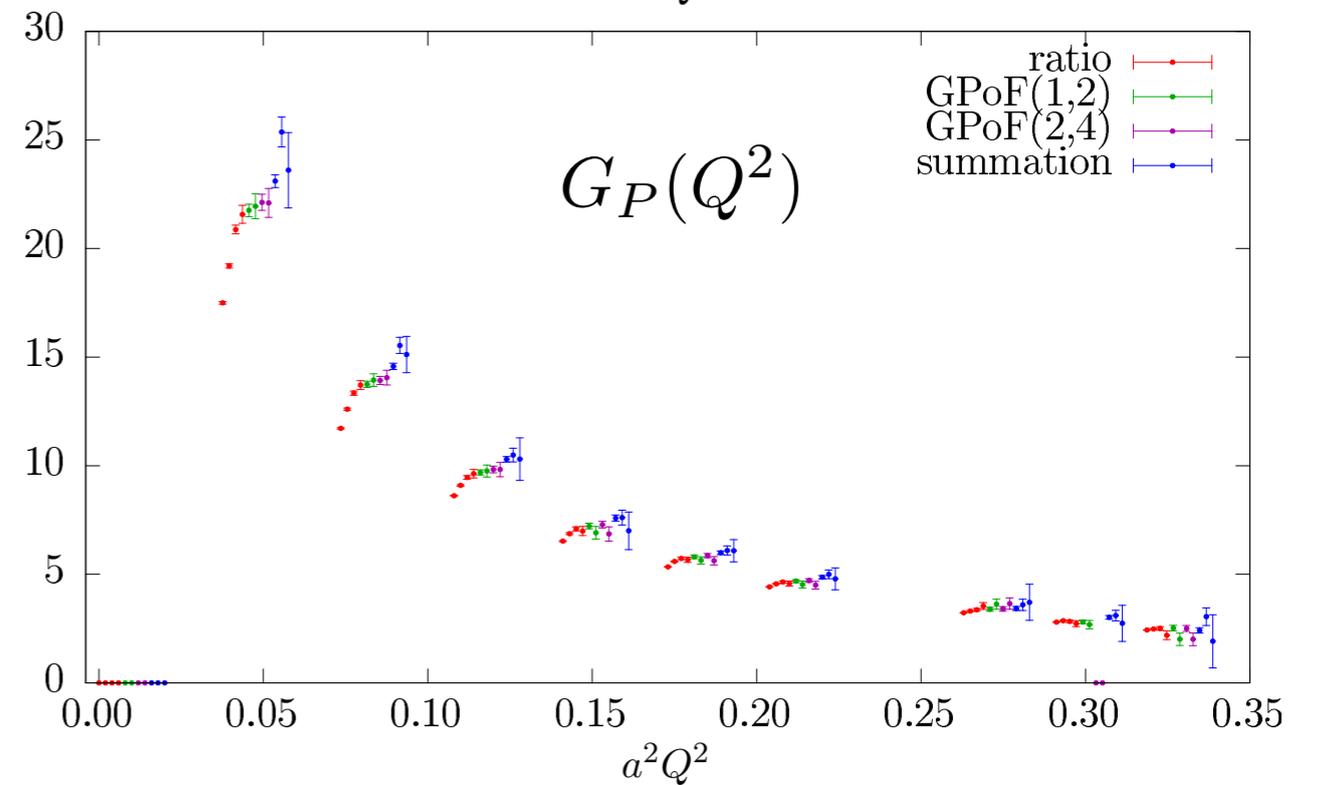
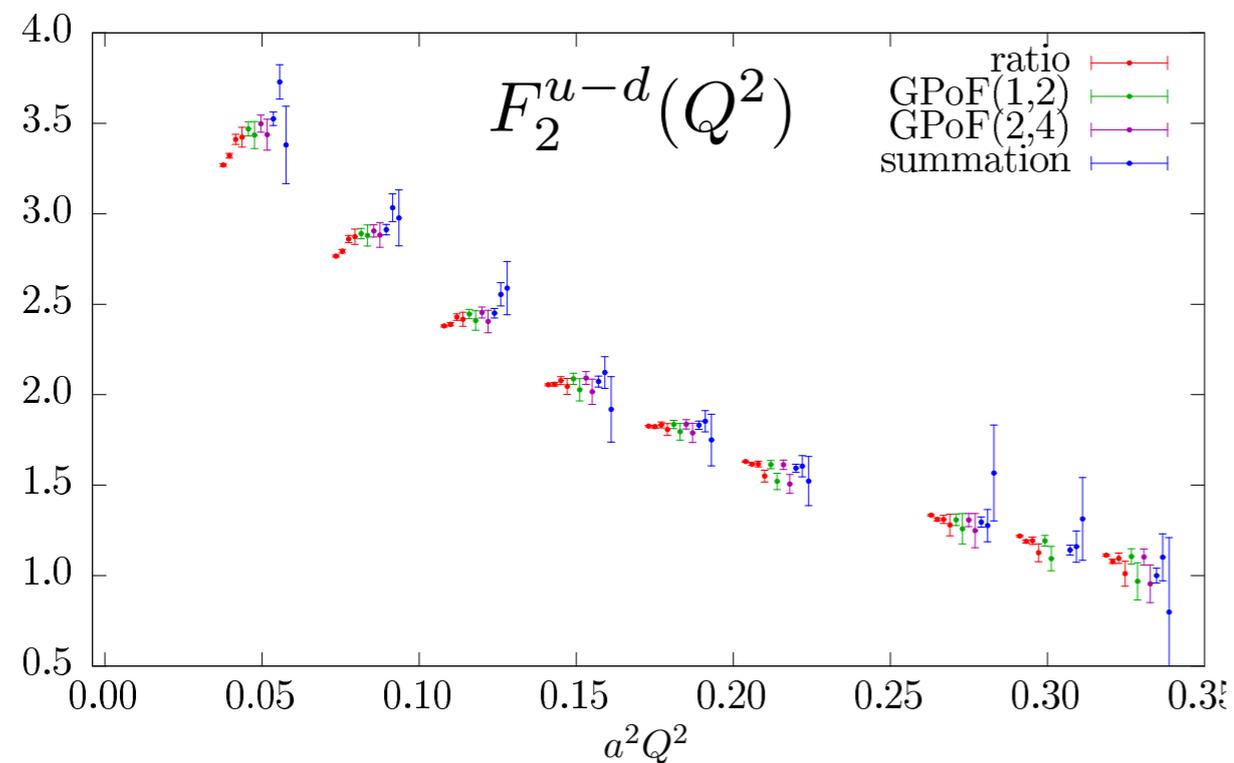
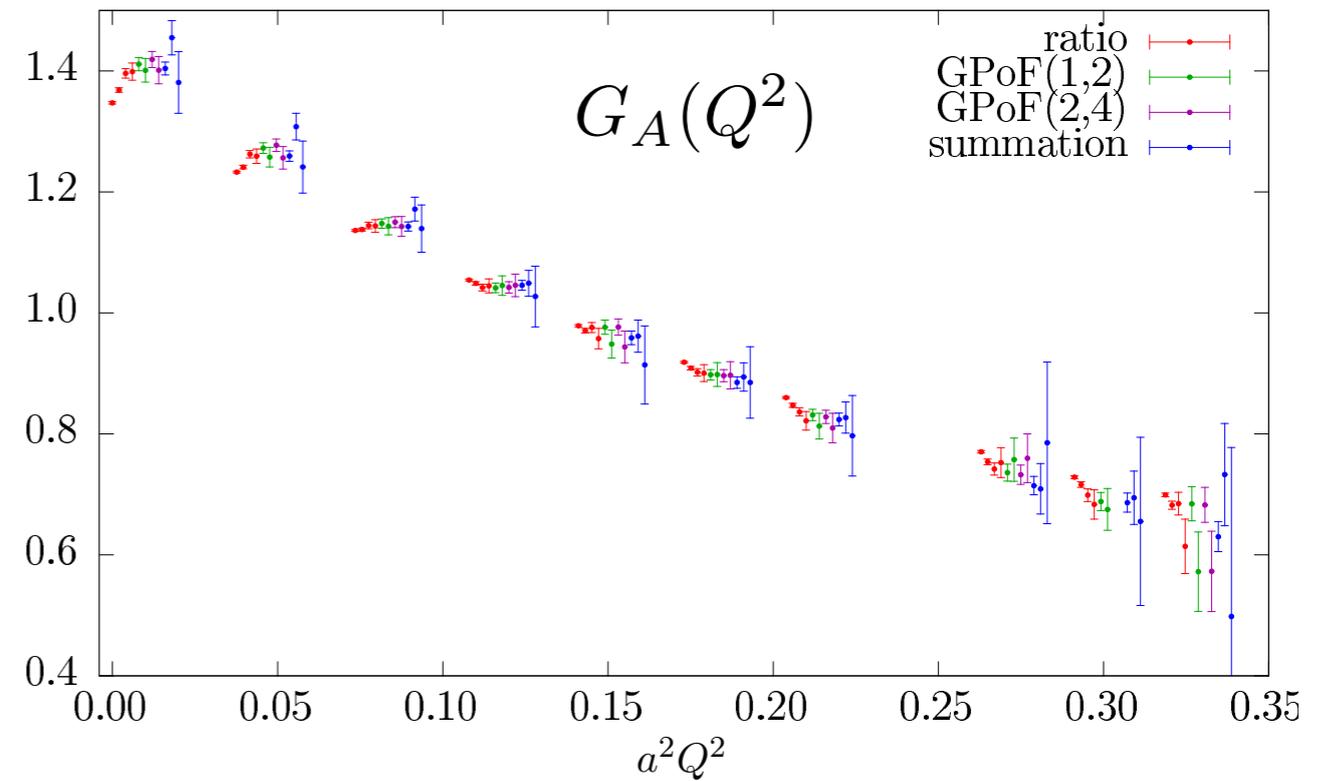
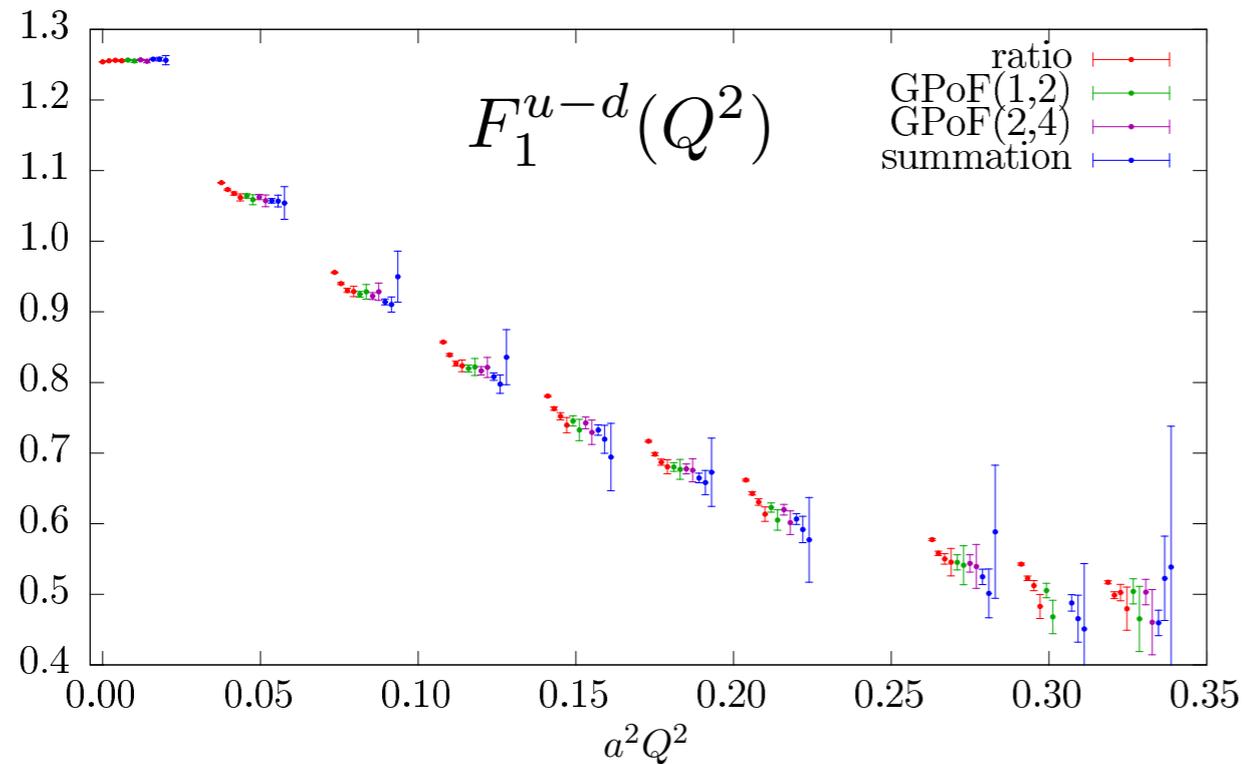


- $G_A(Q^2)$ are measured in ν -scattering, π -production; implications for neutrino flux norm. in IceCube, etc
- Axial radius (r_A^2) = $12 / m_A^2$: model dependence
varying nuclear / G_A shape models: $m_A = 0.9 \dots 1.4$ GeV
- Strange quark $G_{A,P}^s(Q^2)$: MiniBooNE
- $G_P(Q^2)$ induced pseudoscalar : μ capture (MuCAP)



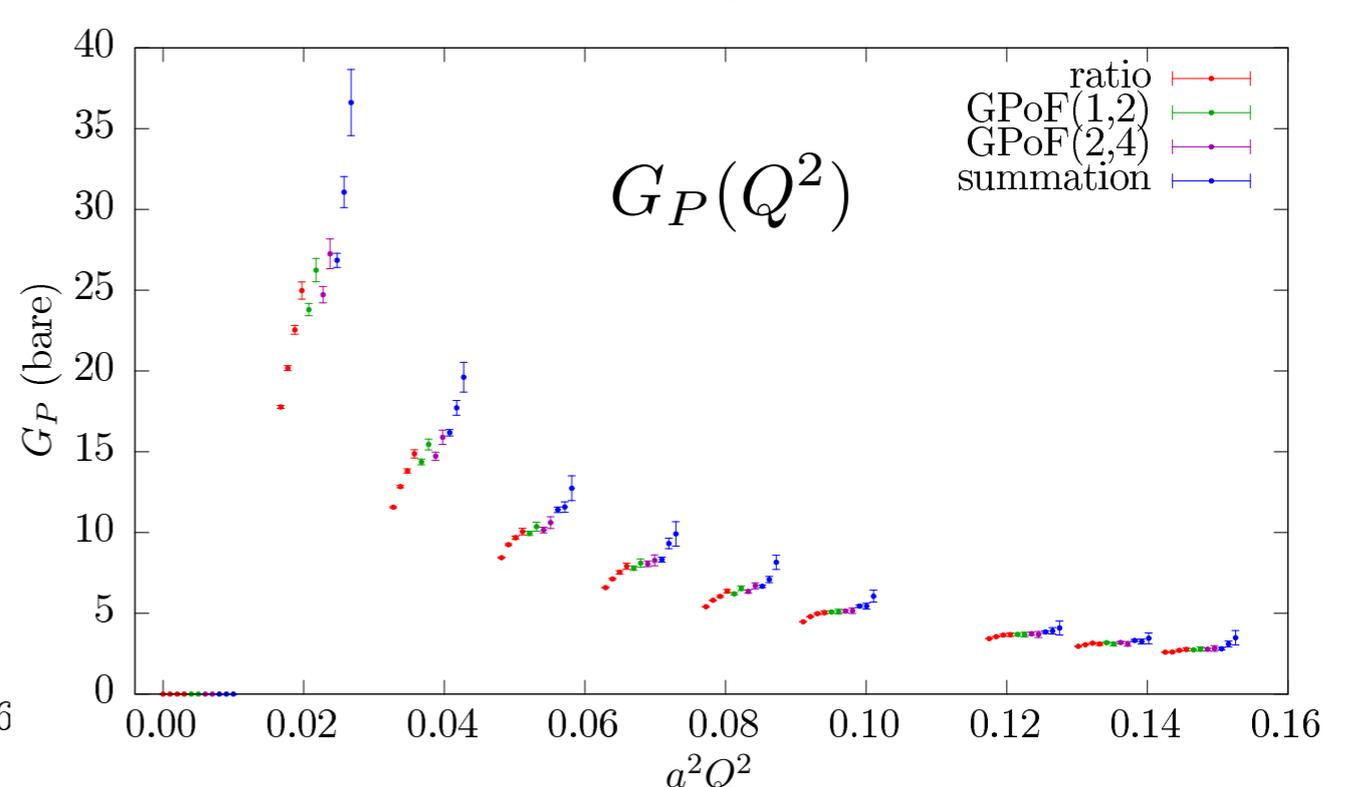
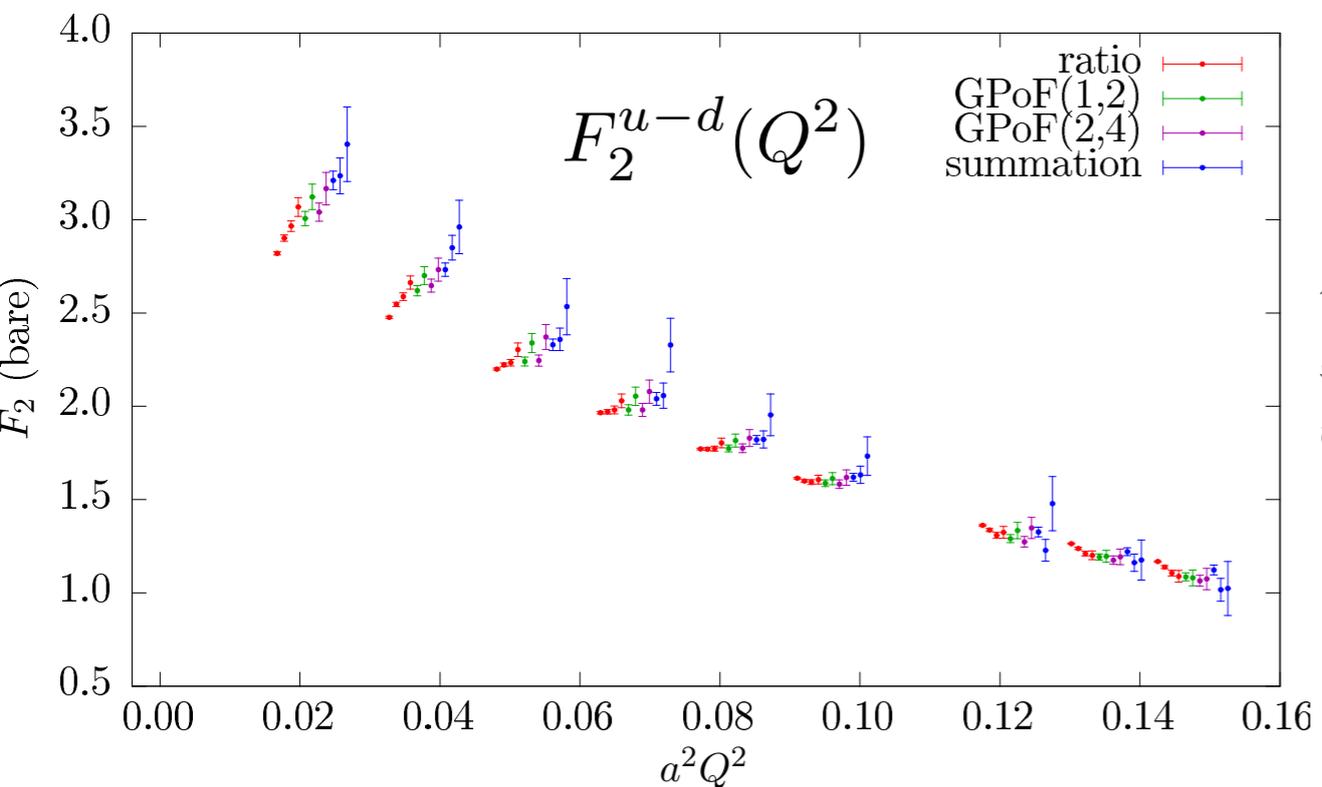
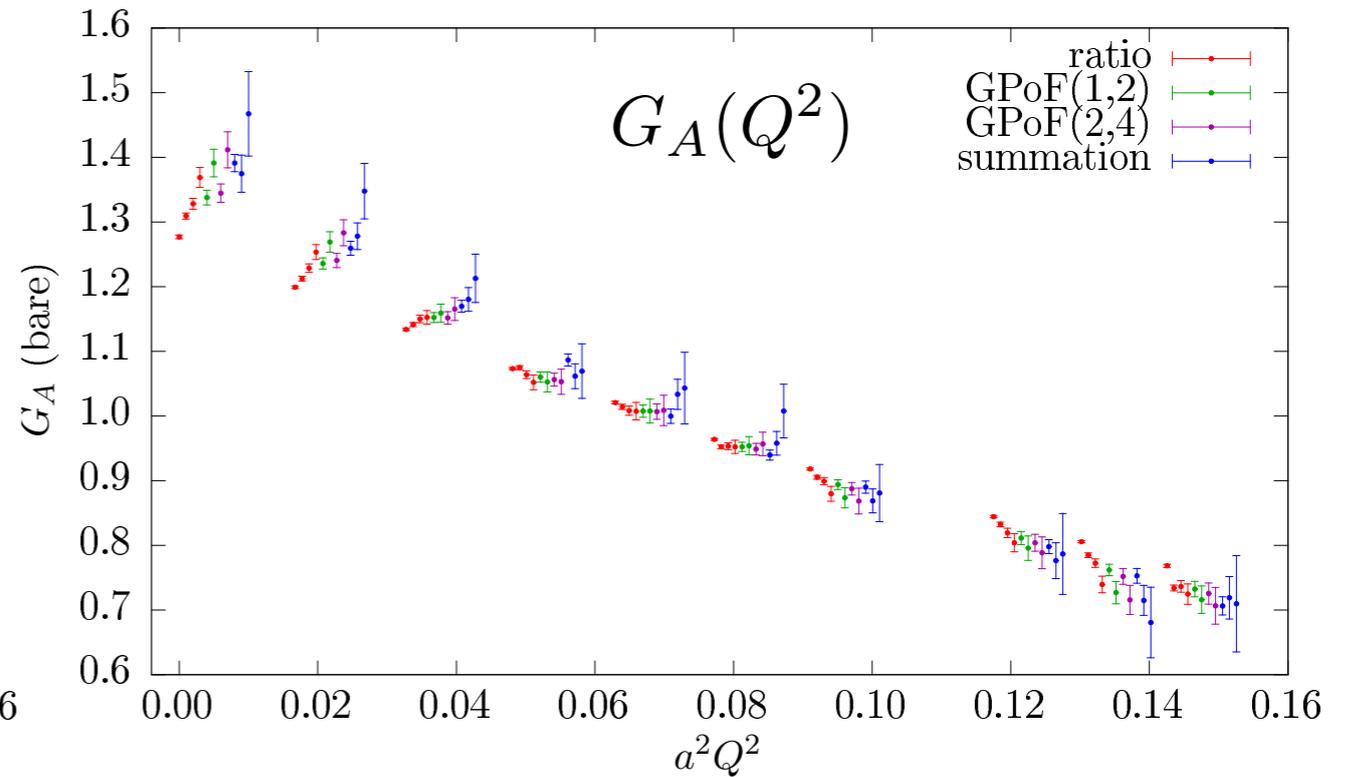
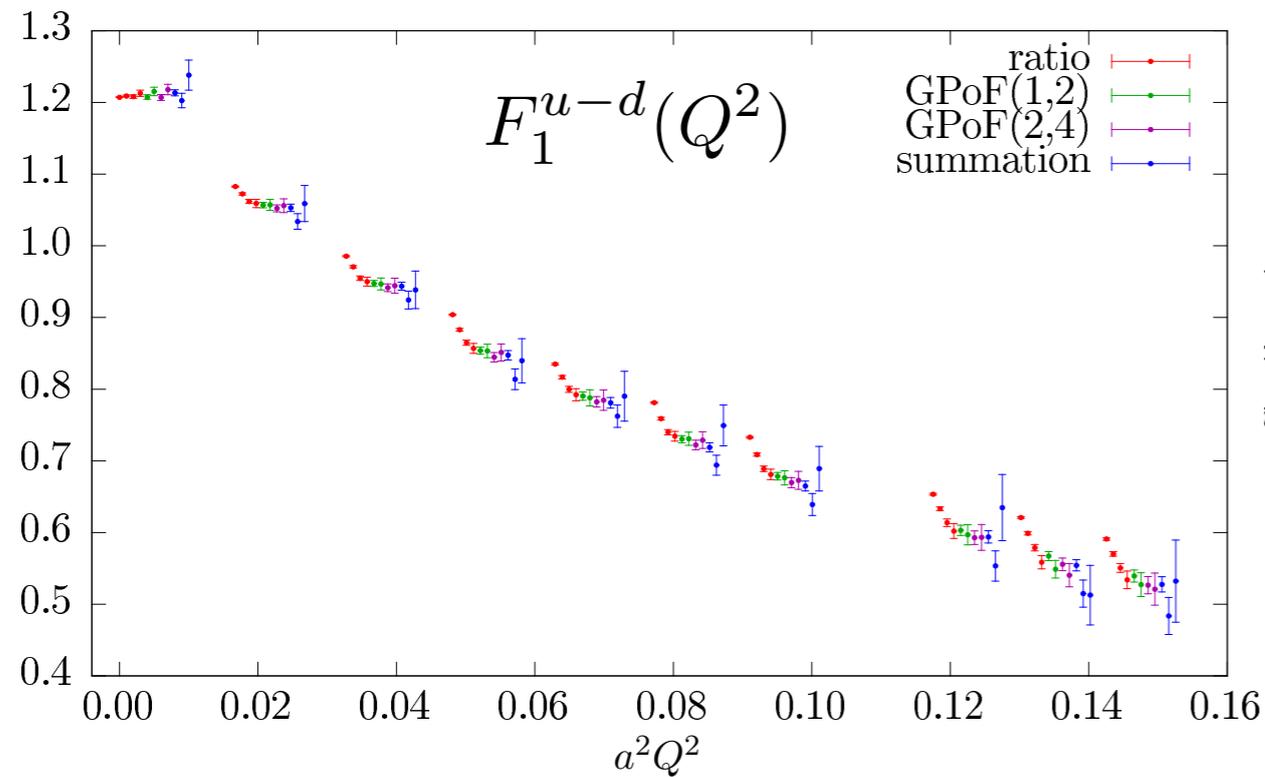
High-precision Form Factors (ALCC 2015-2016)

PRELIMINARY ANALYSIS ($m_\pi=300$ MeV)



High-precision Form Factors (ALCC 2015-2016)

PRELIMINARY ANALYSIS ($m_\pi=190$ MeV)



High-Momentum Nucleon States and Form Factors

- Nucleon operator on a lattice with Gaussian-"smeared" quarks does not couple well to moving hadron

$$N_{\text{lat}}(x) = (\mathcal{S} u)_x^a [(\mathcal{S} d)_x^b C \gamma_5 (\mathcal{S} u)_x^c] \epsilon^{abc}$$

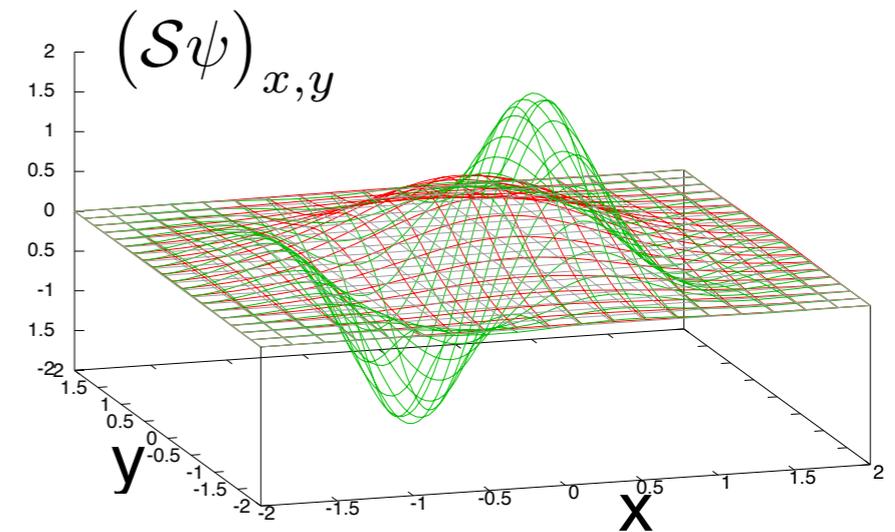
$$\mathcal{S}_{\text{at-rest}} = \exp\left[-\frac{w^2}{4} (i\vec{\nabla})^2\right] \sim \exp\left(-\frac{w^2 \vec{k}_{\text{lat}}^2}{4}\right)$$

reduced overlap with boosted WF

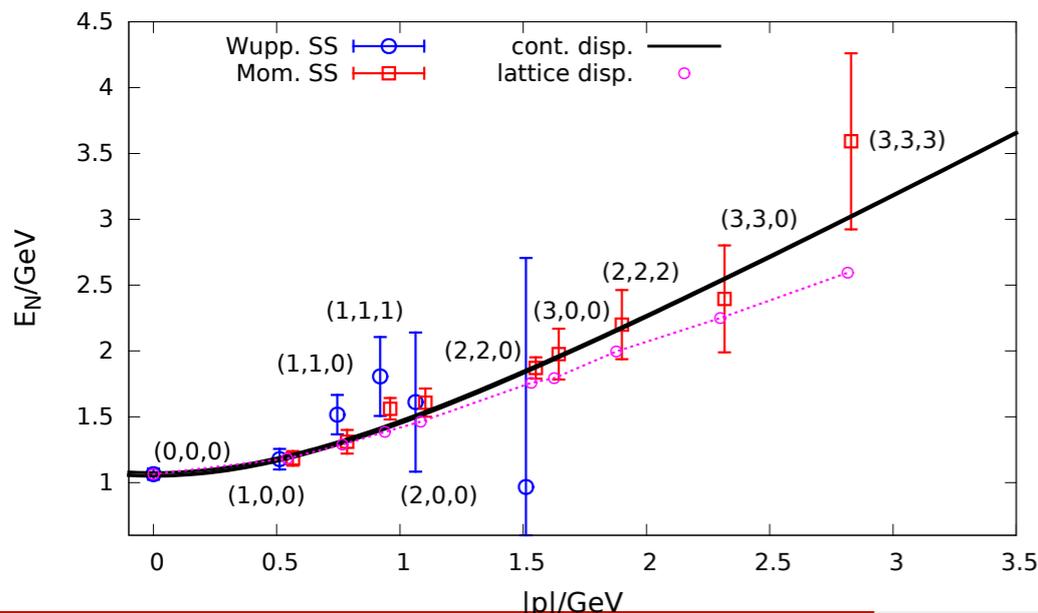
- Optimize smearing for boosted nucleon states [orig. B.Musch]

$$\mathcal{S}_{\text{boosted}} = \exp\left[-\frac{w^2}{4} (i\vec{\nabla} - \vec{k}_0)^2\right]$$

$$\sim \exp\left(-\frac{w^2 (\vec{k}_{\text{lat}} - \vec{k}_0)^2}{4}\right)$$



RQCD results for spectrum [G. Bali et al, arXiv:1602.05525]



This Proposal (**CONN3PT**): study boosted sources with $m_\pi = 320, 190$ MeV with $a=0.114, a=0.081$ fm
In Breit frame:

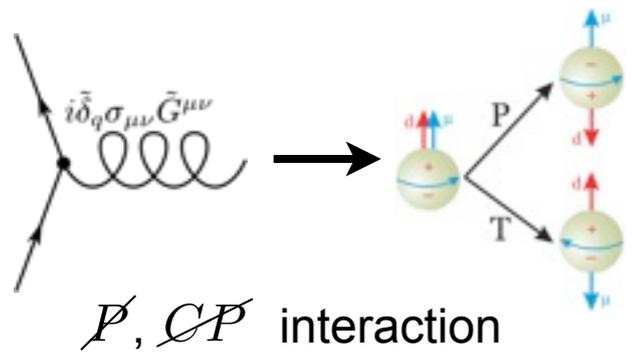
- periodic BC $Q_{\text{opt}}^2 = (6 \vec{k}_{\text{min}})^2 = 4.2 \dots 8.2 \text{ GeV}^2$
- antiperiodic (twisting) $Q_{\text{opt}}^2 = (6 \vec{k}_{\text{min}})^2 = 1.1 \dots 2.1 \text{ GeV}^2$

+ Include disconnected diagrams (**DISCO**)

Motivation : JLab @12 GeV will measure proton, neutron form factors up to $Q^2 = 12..18 \text{ GeV}^2$

Neutron EDM induced by Quark Chromo-EDM

- Neutron EDM is the main direction to search for BSM CP-violation; CP-violating quark-gluon interaction is a potential source of neutron EDM:



$$\langle N | V_\mu(q) | N \rangle = \bar{u}_N \left[\gamma_\mu F_1(q^2) + i \frac{[\gamma_\mu, \gamma_\nu]}{2} q_\nu \frac{F_2(q^2)}{2m_N} + (2im_N \gamma_5 q_\mu - \gamma_\mu \gamma_5 q^2) \frac{F_A(q^2)}{m_N^2} + \frac{[\gamma_\mu, \gamma_\nu]}{2} q_\nu \gamma_5 \frac{F_3(q^2)}{2m_N} \right] u_N$$

\not{P}, \not{CP} interaction \not{P} anapole form factor \not{P}, \not{CP} EDM form factor

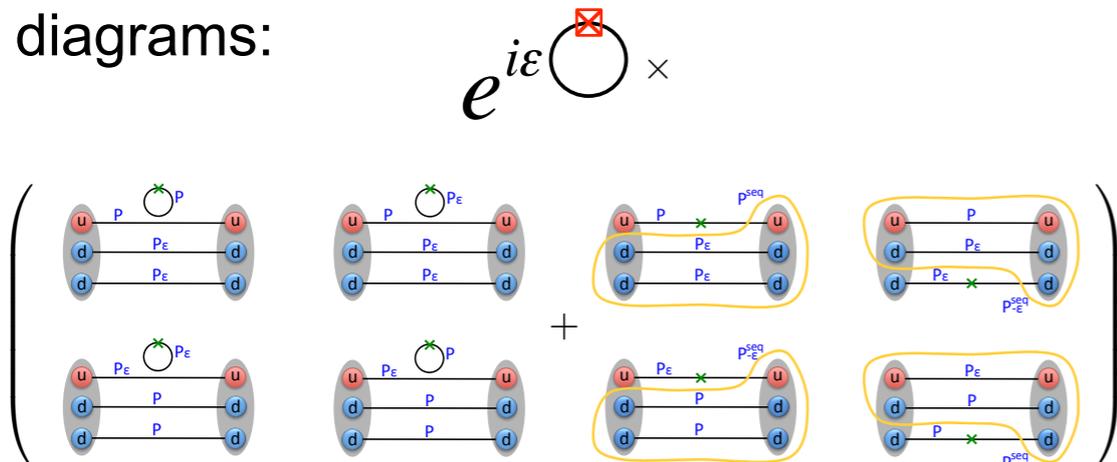
- Schwinger method: Dirac equation modified by chromo-EDM interaction,

$$\left(\not{D} + m - \frac{r}{2} D^2 + c_{sw} \Sigma^{\mu\nu} G_{\mu\nu} \right)^{-1} \rightarrow \left(\not{D} + m - \frac{r}{2} D^2 + \Sigma^{\mu\nu} (c_{sw} G_{\mu\nu} + i\epsilon \tilde{G}_{\mu\nu}) \right)^{-1}$$

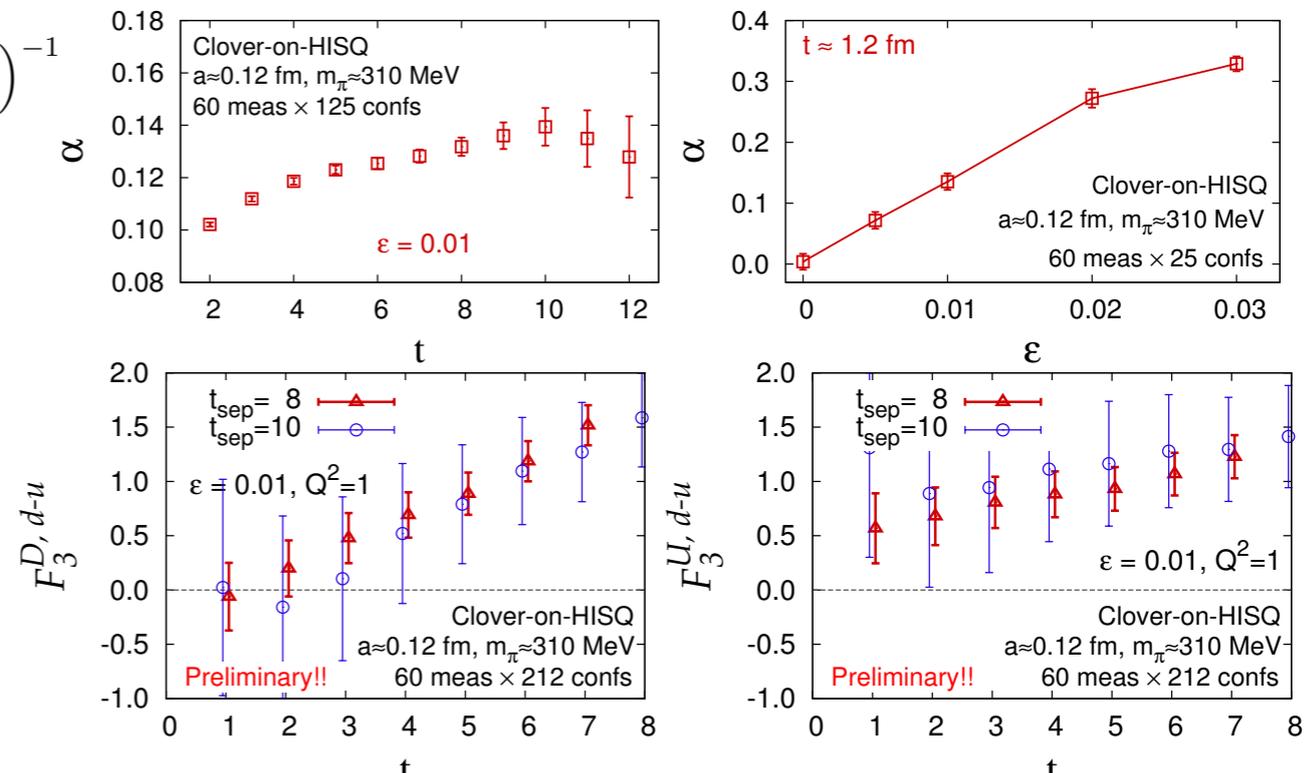
unmodified propagators as initial guess :

Accuracy	$\epsilon = 0.005$	$\epsilon = 0.01$
10^{-8}	85%	86%
10^{-3}	51%	66%
5×10^{-3}	28%	45%

- Complete calculation requires the following diagrams:



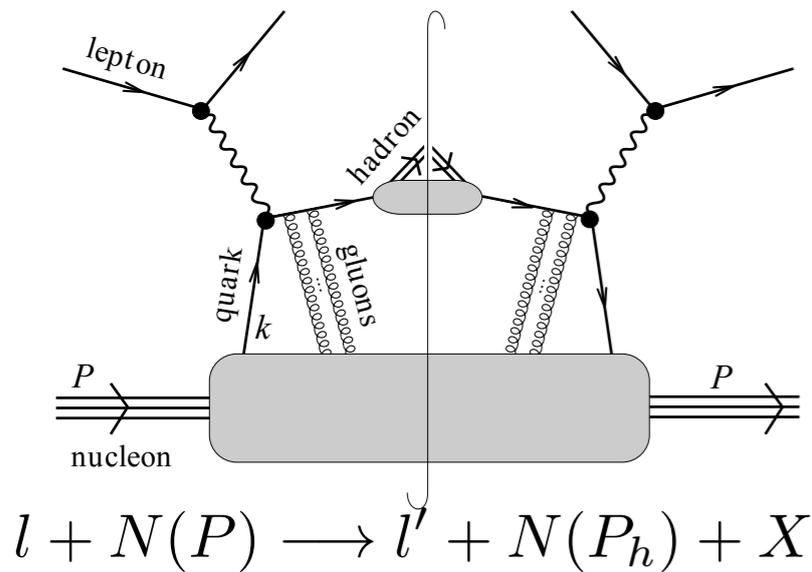
Preliminary results, connected only [T.Bhattacharya, LAT2015]



This proposal: extend to lighter pions, add disconnected diagrams

Transverse Momentum-Dependent Distributions

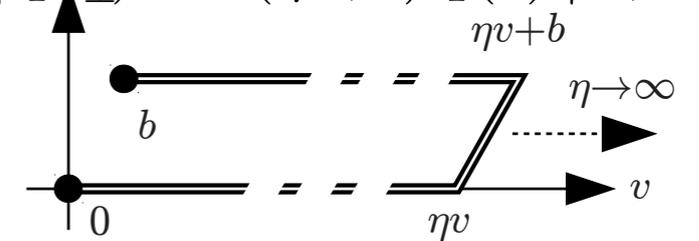
SIDIS



Non-local lattice operator

$$\Phi(b, P, S, \hat{\zeta}, \mu) = \frac{1}{2} \langle P, S | \bar{q}(0) \Gamma \mathcal{U}(\eta v, b) q(b) | P, S \rangle$$

with spacelike link path $\mathcal{U} =$



probes k_{\perp} -moments
("shifts") of TMDs

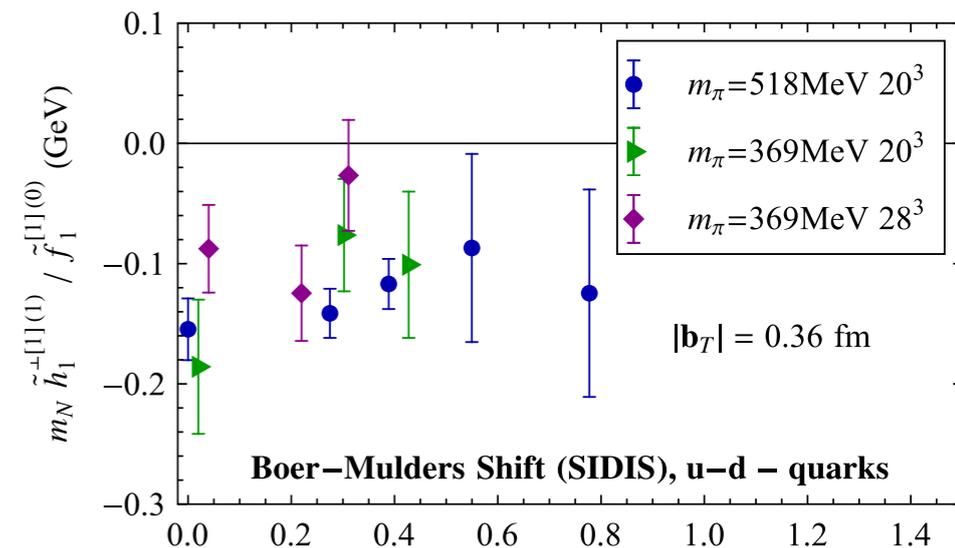
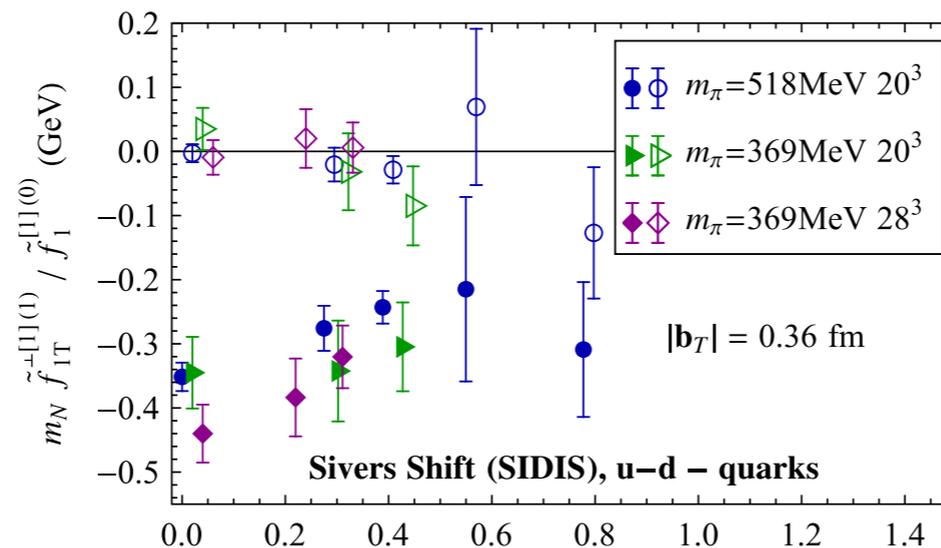
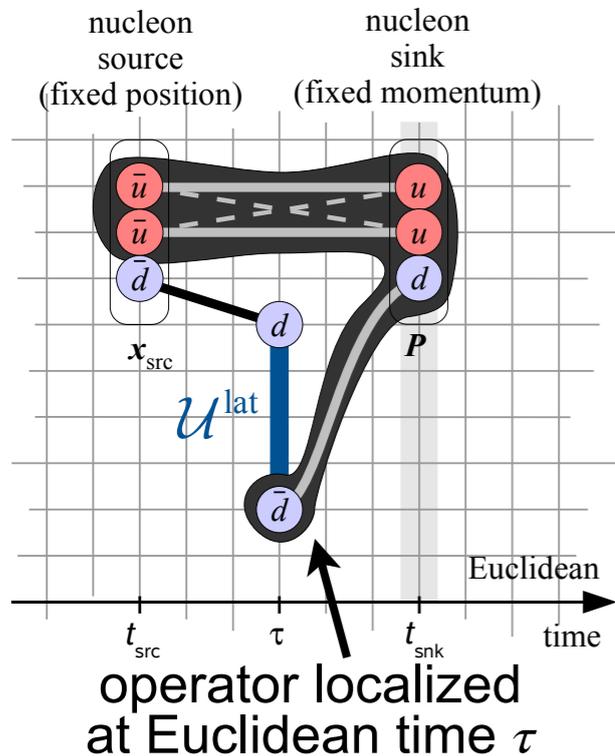
$$\sim \int dx \int d^2 \vec{k}_{\perp} k_i f(x, \vec{k}_{\perp})$$

"light-cone" limit is required
in Collins-Soper parameter

$$\hat{\zeta} = \frac{P \cdot v}{m_N |v|} \rightarrow \infty$$

domain wall on Asqtad

[M.Engelhardt, B.Musch, et al, Few Body Syst (2015)]



Proposed quasi-PDF operators [X.Ji (2013)] are a special case of TMD operators

Total Request

- [DISCO] disconnected quark loops with HP and deflation
- [CONN3PT] form factors at high momentum transfer
- [CEDM] nucleon EDM induced by quark chromo EDM
- [TMD] TMD and PDF contractions for high-momentum nucleon in- & out-states

	C13 : 32 ³ x96 <i>m</i> π =300 MeV a=0.114 fm	D5 : 32 ³ x64 <i>m</i> π =300 MeV a=0.080 fm	D6 : 48 ³ x96 <i>m</i> π =190 MeV a=0.080 fm	REQUEST
DISCO	500c * 512 vec	500c * 512 vec	500c * 512 vec	27.4 M
CONN3PT	38,400 samp.	28,800 samp.	38,400 samp.	15.4 M
CEDM	64,000 samp.	48,000 samp.	64,000 samp.	23.7 M
TMD(contract)		14,400 samp.		12.0 M
TOTAL				86.3 M

SPC Questions

1. With the new resources at JLab being as yet unspecified, we would like to know if you are in a position to use them efficiently if they are a) cpu, b) GPU, c) KNL.

Our project uses Chroma and Qlua suites

- Chroma has multigrid solvers efficient on both CPU and GPU
- Qlua has the same multigrid inverter for CPU [QOPQDP, J.Osborn]
- Members of the team are part of a NESAP project to develop a multi-grid solver for KNL motivated by the commitment to KNL by major research centers including LANL and NERSC.
- Contractions in Chroma will be accelerated with QDP++JIT (needs Intel LLVM compiler, expected Fall 2016); Qlua relies on OpenMP version of QDP/C

*2. What are the prospects for a physical mass clover ensemble?
Are the trajectories listed in proposal thermalized?*

- The two physical point ensembles are planned for generation in the BlueWaters proposal : $72^3 \times 196$ (thermalized towards the physical point now, 5,000 planned in the BW 1st year), and $96^3 \times 256$ (2,000 planned in the BW 2nd year)
- Quoted configuration numbers for proposed calculations are fully thermalized

SPC Questions

3. In the discussion of the axial FF, you mention dipole fits. Do you anticipate using more robust z-parametrisation as in some of your previous work?

- We will consider multiple parameterizations to study how they affect the outcome, including the dipole and the z-expansion.
- The dipole form was highlighted as the one used most often by experimentalists.
- The z-expansion fit has proved very useful for extracting low- Q^2 behavior, such as the axial radius, an important quantity that connects lattice QCD with ChPT.

4. Can you more clearly outline the interconnectedness and dependencies of the various parts of your proposed calculations?

- Efficient calculation of disconnected diagrams is at the center of the proposal
- Neutron EDM will use disconnected insertions of both quark current and chromo-EDM
- High-momentum nucleon structure will use quark-bilinear insertions with high momentum
- Nucleon sources and propagators optimized for high momentum will be used to study the large momentum (Collins-Soper) limit of TMDs & PDFs on a lattice

SPC Questions

5. For quasi-PDFs, there is a competing proposal from Huey-Wen Lin and other collaborators. What is the need and what are the unique features for the quasi-PDF part of that proposal and this current proposal?

The theoretical frameworks for connecting extended lattice operators to PDF are completely new and various lattice approaches are warranted in order to evaluate them. We believe that our proposed calculations are complementary to those proposed by Lin and Collaborators.

- The high-momentum nucleon states on a lattice offer a unique opportunity to study the large-momentum limit that has to be taken for both TMDs and PDFs. PDFs, which are a special case of our TMD contractions, are essentially free in our calculation
- The Huey-Wen's proposal with similar pion mass but with much finer lattice spacing, $a=0.045$ fm, to study the systematics due to lattice artifacts $(pa)^n$. Additionally, they plan to look at the strange and charm PDFs, to investigate how well the LaMET framework (Jiunn-Wei Chen, Jianhui Zhang, Xiangdong Ji) applies to heavier quarks, and make connections to the global fits (C.-P. Yuan).

SPC Questions

6. Since you are planning to calculate g_A for which there already exists an accurate measurement, have you considered performing a “blind analysis” to prevent any inadvertent bias? To blind your analysis, you could add an overall off-set factor to the correlation functions that would be kept unknown to the people doing the analysis until the systematic error analysis is finalized.

There is indeed the need to implement robust measures into the analysis to prevent human bias
(*Thanks for the encouragement and starting the discussion!*)

- Data analysis has been streamlined, with the analysis stage separated from "stripping" the raw output; adding quantity-dependent offset factors is trivial.
- Random variation to offset factors ("artificial noise") may be helpful to evaluate our separation of the stochastic noise from systematic uncertainties (the latter should be stable when the artificial noise is removed)